Two-body state with *p*-wave interaction in a one-dimensional waveguide under transversely anisotropic confinement

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We theoretically study two atoms with *p*-wave interaction in a one-dimensional waveguide, investigating how the transverse anisotropy of the confinement affects the two-body state, especially the properties of the resonance. For a bound-state solution, we find there are a total of three two-body bound states due to the richness of the orbital magnetic quantum number of the *p*-wave interaction, while only one bound state is supported by the *s*-wave interaction. Two of them become nondegenerate due to the breaking of the rotation symmetry under a transversely anisotropic confinement. For a scattering solution, the effective one-dimensional scattering amplitude and scattering length are derived. We find the position of the *p*-wave confinement-induced resonance shifts apparently versus the transverse anisotropy. In addition, a two-channel mechanism for the confinement-induced resonance in a one-dimensional waveguide is generalized to the *p*-wave interaction, which was previously proposed only for the *s*-wave interaction. All our calculations are based on the parametrization of the ^{40}K-atom experiments and can thus be confirmed in future experiments.

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I. INTRODUCTION

Nowadays, confinement-induced resonance (CIR) as one of the most intriguing phenomena in low-dimensional systems has attracted a great deal of interest. It was first predicted by Olshanii, who considered the two-body *s*-wave scattering problem in quasi-one-dimensional (quasi-1D) waveguides [1]. Subsequently, this study was extended to quasi-two-dimensional (quasi-2D) systems [2,3]. In the past decade, an impressive amount of experimental and theoretical efforts have been devoted to confirm the existence of CIRs and to explore important consequences [4–18]. To date, CIR has already become a fundamental technique in studying strongly interacting low-dimensional quantum gases.

The *p*-wave interaction is of particular interest, since it is the simplest high-partial-wave interaction with nonzero orbital angular momentum $l = 1$. This leads to a spatial anisotropic scattering in few-body physics, opening a new method to manipulate resonant cold atomic interactions by using a magnetic field vector, as indicated in our previous works [18]. For many-body physics, the high-partial-wave interaction may also result in physically abundant quantum-phase transitions, which are absent in the *s*-wave interaction [19–22]. In addition, the *p*-wave scattering dominates the low-energy interaction between spin-polarized Fermi atoms (all in one hyperfine state) due to Fermi-Dirac statistics, which provides an ideal candidate to study the *p*-wave scattering properties in cold atoms [23,24].

In low-dimensional quantum systems, the external trap potential is one another degree of freedom to manipulate two-body resonant interactions, where it is interesting to identify how CIR is affected by the external confinement. For example, in 1D Bose ^{133}Cs atoms with the *s*-wave interaction, it is found that the transverse harmonic anisotropy shifts the position of CIR [12,13], and CIR splits due to the anharmonicity of the transverse trap [10,14,15]. Naturally, it is also important to investigate how the external confinement affects the low-dimensional resonant *p*-wave interaction between spin-polarized Fermi atoms, such as ^{40}K atoms.

The *p*-wave scattering problem in quasi-1D waveguides was previously studied under a transversely isotropic confinement, by using the *K* matrix [5] and pseudopotential methods [8], respectively. In this paper, we directly generalize Pricoupenko's theory [8] to study the two-body problem with *p*-wave interaction in quasi-1D waveguides under a transversely anisotropic confinement, and we investigate the influence of the transverse anisotropy on the two-body state, especially the scattering resonance in quasi-1D waveguides. The *p*-wave interaction is modeled by the contact pseudopotential including the effective range [8,25]. This model neglects the spatial anisotropy of the *p*-wave interaction, which is not the focus of this work. However, it is already sufficient to study how the external trap affects the *p*-wave CIR. Compared to Ref. [8], our theory is a useful generalization in obtaining the following results:

(1) Due to the nonzero orbital angular momentum, there are a total of three bound states for the *p*-wave interaction in the waveguide, while the *s*-wave pseudopotential supports only one two-body bound state. Two of them become nondegenerate as the transverse confinement changes from isotropic to anisotropic, which breaks the rotation symmetry around the waveguide.

(2) For the scattering solution, we calculate the effective 1D scattering amplitude as well as the 1D scattering length, and we find the resonant position apparently shifts from Pricoupenko's result as the transverse anisotropy increases, which can become a technique to manipulate the low-dimensional *p*-wave interaction. Our results can recover Pricoupenko's results when transverse confinement becomes isotropic.

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(3) We present an effective two-channel mechanism for the p-wave CIR, which was first proposed only for the s-wave interaction [4]. In this two-channel picture, the transverse ground state and the remaining transverse excited modes play roles of open and closed channels, respectively. When a bound state in the closed channel exists, and becomes energetically degenerate with the scattering threshold of the open channel, a scattering resonance is expected. The two-channel description of the p-wave CIR agrees with that of the scattering-amplitude calculation. All our calculations are based on the parametrization of the ^{40}K-atom experiments [7,24] and can be confirmed in future experiments.

In the following, we first study properties of two-body bound states with p-wave interaction in a transversely anisotropic waveguide (Sec. II), and then we present the scattering solution in Sec. III as well. We calculate the effective 1D scattering amplitude and scattering length, discussing how the transverse anisotropy of the confinement affects the p-wave CIR in detail. In Sec. IV, a two-channel mechanism is presented for the p-wave CIR. The main results are concluded in Sec. V.

II. BOUND-STATE SOLUTIONS

In order to simplify the problem, let us consider two atoms confined in a quasi-1D waveguide with a tight transverse harmonic potential (in the x-y plane). The transverse trap frequencies are $\omega_{x,y}$, and the atoms can freely move along the z axis. The transverse anisotropy is characterized by $\eta = \omega_x/\omega_y$. Unlike the s-wave interaction, the p-wave interaction is strongly dependent on energy due to its narrow-width property [24]. Thus, one additional parameter, effective range, should be included [25–28]. In harmonic traps, the center-of-mass (c.m.) motion is decoupled from the relative part, so we only need to solve the relative motion, while the c.m. motion is a simple harmonic oscillator. By dropping the c.m. motion off, the relative motion of two atoms is described by the following Hamiltonian:

$$\mathcal{H} = -\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial z^2} + \mathcal{H}_\perp + \mathcal{V}_1, \tag{1}$$

where

$$\mathcal{H}_\perp = -\frac{\hbar^2}{2\mu}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) + \frac{1}{2}\mu\omega^2(\eta^2 x^2 + y^2), \tag{2}$$

μ is the reduced mass, and ω is the trap frequency in the y axis (we omit the subscript y without ambiguity). The interatomic interaction $\mathcal{V}_1(\mathbf{r})$ is modeled by the p-wave pseudopotential [25],

$$\mathcal{V}_1(\mathbf{r}) = \frac{\pi\hbar^2}{\mu}\left(\frac{1}{v_1} - \frac{1}{2}r_1 k^2\right)^{-1} \overleftarrow{\nabla}\delta(\mathbf{r})\frac{\partial^3}{\partial r^3}r^3\overrightarrow{\nabla}, \tag{3}$$

where v_1 and r_1 are the three-dimensional (3D) p-wave scattering volume and effective range (with a dimension of inverse length), respectively, which can be tuned by using p-wave Feshbach resonances [23,24]. k is the relative wave number, related to the relative energy of two atoms as $E = \hbar^2k^2/(2\mu)$. The symbol $\overleftarrow{\nabla}$ ($\overrightarrow{\nabla}$) denotes the gradient operator that acts to the left (right) of the pseudopotential.

For the bound-state problem, the wave function can formally be written as

$$\psi(\mathbf{r}) = -\int d^3\mathbf{r}' G_E(\mathbf{r},\mathbf{r}')\mathcal{V}_1(\mathbf{r}')\psi(\mathbf{r}'), \tag{4}$$

where $G_E(\mathbf{r},\mathbf{r}')$, the single-particle Green's function with energy E, satisfies

$$\left(-\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial z^2} + \mathcal{H}_\perp - E\right)G_E(\mathbf{r},\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}'). \tag{5}$$

The Green's function can be expanded in series of the eigenstates of the noninteracting Hamiltonian as

$$G_E(\mathbf{r},\mathbf{r}') = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\int_{-\infty}^{\infty} dk_z C_{n_1n_2}(k_z)\phi_{n_1}\left(\frac{\sqrt{\eta}x}{d}\right)\phi_{n_2}\left(\frac{y}{d}\right)\frac{e^{ik_z z}}{\sqrt{2\pi}}, \tag{6}$$

where $\phi_\nu(\cdot)$ is the eigenstate of the 1D harmonic oscillator, $d = \sqrt{\hbar/\mu\omega}$ is the harmonic length in the y axis, and k_z is the wave number along the waveguide (z axis). Inserting Eq. (6) into Eq. (5) and using the completeness of the eigenstates of the noninteracting Hamiltonian, we may easily obtain the coefficients $C_{n_1n_2}(k_z)$, and then after some straightforward algebra (the derivation is similar to Eq. (24) in [12]), the Green's function takes the following integral representation:

$$G_E(\mathbf{r},\mathbf{r}') = \frac{1}{\pi^{3/2}d^3\hbar\omega}\int_0^\infty \frac{d\tau\, e^{\epsilon\tau}}{\sqrt{2\tau}}\exp\left[-\frac{(z-z')^2}{2d^2\tau}\right]\frac{\sqrt{\eta}}{\sqrt{1-e^{-2\eta\tau}}}\exp\left[\eta\frac{2x'x-(x'^2+x^2)\cosh(\eta\tau)}{2d^2\sinh(\eta\tau)}\right] \times\frac{1}{\sqrt{1-e^{-2\tau}}}\exp\left[\frac{2y'y-(y'^2+y^2)\cosh\tau}{2d^2\sinh\tau}\right], \tag{7}$$

which is valid for $\epsilon \equiv E/\hbar\omega - (\eta+1)/2 < 0$. Combining Eqs. (3) and (4), the bound-state wave function takes the form

$$\psi(\mathbf{r}) = -\frac{\pi\hbar^2}{\mu}\left(\frac{1}{v_1} - \frac{1}{2}r_1k^2\right)^{-1}\sum_n \mathcal{R}_n\left[\frac{\partial}{\partial r_n'}G_E(\mathbf{r},\mathbf{r}')\right]_{r'=0}, \tag{8}$$

where the summation is over $n = x,y,z$, and $r_x \equiv x$, $r_y \equiv y$, $r_z \equiv z$. Here, we have defined the coefficient

$$\mathcal{R}_n \equiv \left[\frac{\partial^3}{\partial r^3}r^3\frac{\partial}{\partial r_n}\psi(\mathbf{r})\right]_{r=0}. \tag{9}$$

Acting $\frac{\partial^3}{\partial r^3} r^3 \frac{\partial}{\partial r_n}$ on both sides of Eq. (8) and setting $r = 0$, we obtain the following secular equation:

$$\sum_n \left[-\frac{\pi\hbar^2}{\mu} \left(\frac{1}{v_1} - \frac{1}{2} r_1 k^2 \right)^{-1} \mathcal{A}_{ln} - \delta_{ln} \right] \mathcal{R}_n = 0 \quad (l = x, y, z), \tag{10}$$

where

$$\mathcal{A}_{ln} \equiv \left[\frac{\partial^3}{\partial r^3} r^3 \frac{\partial^2}{\partial r_l \partial r_n'} G_E(\mathbf{r}, \mathbf{r}') \right]_{r=r'=0}. \tag{11}$$

For any nonzero vector $\mathbf{r}$, it is easy to show that

$$\left[\frac{\partial G_E(\mathbf{r}, \mathbf{r}')}{\partial r_n'} \right]_{r'=0} = \frac{r_n}{\pi^{3/2} d^5 \hbar\omega} \left[\mathcal{F}_n(\epsilon, \mathbf{r}) + \frac{\sqrt{\pi}}{2} \left(\frac{d^3}{r^3} + \frac{E}{\hbar\omega} \frac{d}{r} \right) \right], \tag{12}$$

in which

$$\mathcal{F}_x(\epsilon, \mathbf{r}) = \int_0^\infty d\tau \left\{ \frac{\eta^{3/2} e^{\epsilon\tau}}{\sqrt{2}\sinh(\eta\tau)} \frac{\exp\left[-\frac{\eta x^2}{2d^2 \tanh(\eta\tau)} - \frac{y^2}{2d^2 \tanh\tau} - \frac{z^2}{2d^2\tau} \right]}{\sqrt{\tau(1 - e^{-2\eta\tau})(1 - e^{-2\tau})}} - \frac{1}{2\sqrt{2}} \left(\frac{1}{\tau^{5/2}} + \frac{E}{\hbar\omega} \frac{1}{\tau^{3/2}} \right) e^{-\frac{r^2}{2d^2\tau}} \right\}, \tag{13}$$

$$\mathcal{F}_y(\epsilon, \mathbf{r}) = \int_0^\infty d\tau \left\{ \frac{\eta^{1/2} e^{\epsilon\tau}}{\sqrt{2}\sinh\tau} \frac{\exp\left[-\frac{\eta x^2}{2d^2 \tanh(\eta\tau)} - \frac{y^2}{2d^2 \tanh\tau} - \frac{z^2}{2d^2\tau} \right]}{\sqrt{\tau(1 - e^{-2\eta\tau})(1 - e^{-2\tau})}} - \frac{1}{2\sqrt{2}} \left(\frac{1}{\tau^{5/2}} + \frac{E}{\hbar\omega} \frac{1}{\tau^{3/2}} \right) e^{-\frac{r^2}{2d^2\tau}} \right\}, \tag{14}$$

$$\mathcal{F}_z(\epsilon, \mathbf{r}) = \int_0^\infty d\tau \left\{ \frac{\eta^{1/2} e^{\epsilon\tau}}{\sqrt{2}\tau} \frac{\exp\left[-\frac{\eta x^2}{2d^2 \tanh(\eta\tau)} - \frac{y^2}{2d^2 \tanh\tau} - \frac{z^2}{2d^2\tau} \right]}{\sqrt{\tau(1 - e^{-2\eta\tau})(1 - e^{-2\tau})}} - \frac{1}{2\sqrt{2}} \left(\frac{1}{\tau^{5/2}} + \frac{E}{\hbar\omega} \frac{1}{\tau^{3/2}} \right) e^{-\frac{r^2}{2d^2\tau}} \right\}. \tag{15}$$

Substituting Eq. (12) into Eq. (11) directly yields

$$\mathcal{A}_{ln} = \frac{6}{\pi^{3/2} d^5 \hbar\omega} \mathcal{F}_n(\epsilon, 0)\, \delta_{ln}, \tag{16}$$

where δ_{ln} is the Kronecker delta function. Combining Eqs. (10) and (16), the vanish of the determinant of the coefficient matrix in Eq. (10) determines the binding energy, which yields

$$\frac{d^3}{v_1} = -\frac{6}{\sqrt{\pi}} \mathcal{F}_l(\epsilon, 0) + d r_1 \frac{E}{\hbar\omega} \quad (l = x, y, z). \tag{17}$$

Obviously, there are a total of three bound states for p-wave interaction, while the s-wave pseudopotential supports only one two-body bound state. This results from the richness of the orbital magnetic quantum number of the p-wave interaction. The corresponding (un-normalized) bound-state wave function is

$$\psi_l(\mathbf{r}) = r_l \left[\frac{1}{r^3} + \frac{E}{\hbar\omega} \frac{1}{r d^2} + \frac{2}{\sqrt{\pi} d^3} \mathcal{F}_l(\epsilon, \mathbf{r}) \right] \quad (l = x, y, z). \tag{18}$$

Obviously, these bound states can be classified into two kinds by the transverse parity (in x or y axes), where $\psi_{x,y}(\mathbf{r})$ has odd transverse parity and $\psi_z(\mathbf{r})$ has even transverse parity. For a transversely isotropic confinement ($\eta = 1$), there is a rotation symmetry around the z axis, and $\psi_x(\mathbf{r})$ and $\psi_y(\mathbf{r})$ are degenerate. Thus, at a small separation, i.e., $\mathbf{r} \approx 0$, we can construct another set of eigenwave functions with specific orbital magnetic quantum number as a superposition of $\psi_x(\mathbf{r})$, $\psi_y(\mathbf{r})$, and $\psi_z(\mathbf{r})$, i.e.,

$$\psi_{m=\pm 1}(\mathbf{r}) = \mp \sqrt{\frac{3}{8\pi}} [\psi_x(\mathbf{r}) \pm i \psi_y(\mathbf{r})], \tag{19}$$

$$\psi_{m=0}(\mathbf{r}) = \sqrt{\frac{3}{4\pi}} \psi_z(\mathbf{r}), \tag{20}$$

or explicitly,

$$\psi_{m=\pm 1}(\mathbf{r}) = \left[\frac{1}{r^2} + \frac{E}{d^2 \hbar\omega} + \frac{2r}{\sqrt{\pi} d^3} \mathcal{F}_x(\epsilon, 0) \right] Y_{1\pm 1}(\hat{\mathbf{r}}), \tag{21}$$

$$\psi_{m=0}(\mathbf{r}) = \left[\frac{1}{r^2} + \frac{E}{d^2 \hbar\omega} + \frac{2r}{\sqrt{\pi} d^3} \mathcal{F}_z(\epsilon, 0) \right] Y_{10}(\hat{\mathbf{r}}), \tag{22}$$

with specific orbital magnetic quantum numbers $m = 0, \pm 1$, where

$$Y_{1\pm 1}(\hat{\mathbf{r}}) = \mp \sqrt{\frac{3}{8\pi}} \frac{x \pm i y}{r}, \tag{23}$$

$$Y_{10}(\hat{\mathbf{r}}) = \sqrt{\frac{3}{4\pi}} \frac{z}{r} \tag{24}$$

are spherical harmonic functions. As the transverse confinement becomes anisotropic, the rotation symmetry around the z axis is broken. This results in the nondegeneracy of $\psi_x(\mathbf{r})$ and $\psi_y(\mathbf{r})$, and they are no longer the superposition of those with specific orbital magnetic quantum number at small $\mathbf{r}$.

We predict the binding energy of two ^{40}K atoms in the $|F = \frac{9}{2}, m_F = -\frac{7}{2}\rangle$ hyperfine state as the function of the magnetic field near the p-wave Feshbach resonance centered at $B_0 = 198.8$ G in three dimensions [7,24], confined in a transversely isotropic waveguide (Fig. 1) as well as in a transversely anisotropic waveguide (Fig. 2). For the isotropic confinement ($\eta = 1$), $\psi_x(\mathbf{r})$ and $\psi_y(\mathbf{r})$ are degenerate as we anticipate (see Fig. 1). But they become distinguishable in energy due to the rotation symmetry breaking around the z axis when the transverse trap becomes anisotropic (see Fig. 2). As the magnetic field increases, these bound states gradually merge into the continuum at different energy thresholds, i.e., $(3\eta + 1)/2$, $(\eta + 3)/2$, and $(\eta + 1)/2$, respectively.

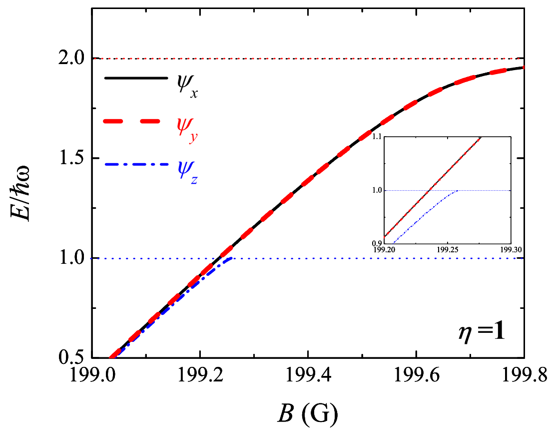

FIG. 1. (Color online) The bound-state energy of two ^{40}K atoms in the $|F=\frac{9}{2},m_F=-\frac{7}{2}\rangle$ hyperfine state confined in a transversely isotropic waveguide near the p-wave Feshbach resonance centered at $B_0=198.8$ G in three dimensions [7,24]. The dotted horizontal lines are the thresholds where the bound states merge into the continuum as the magnetic field increases. The magnetic field B dependence of the scattering volume v_1 as well as the effective range r_1 for ^{40}K is from Ref. [24]. The inset shows the details of the curves in a smaller magnetic field range for visual clarity.

III. SCATTERING SOLUTIONS AND CONFINEMENT-INDUCED RESONANCES

In this section, we generalize Pricoupenko's quasi-1D scattering theory [8] to the case under a transversely anisotropic confinement. Let us consider the p-wave scattering problem in the 1D waveguide with energy just above the transverse zero-point energy, i.e., $\epsilon\to 0^+$. According to the Lippman-Schwinger equation, the scattering solution of the Hamiltonian (1) can formally be written as

$$\psi(\mathbf{r})=\psi_0(\mathbf{r})-\int d^3\mathbf{r}' G_E(\mathbf{r},\mathbf{r}')\mathcal{V}_1(\mathbf{r}')\psi(\mathbf{r}'), \tag{25}$$

where $\psi_0(\mathbf{r})$ is the incident wave function. Since the energy of the relative motion is just above the transverse zero-point energy, the atoms should enter from the transverse ground state, and the incident wave function takes the form of

$$\psi_0(\mathbf{r})=\phi_0\left(\frac{\sqrt{\eta}x}{d}\right)\phi_0\left(\frac{y}{d}\right)i\sin(k_z z). \tag{26}$$

Here, we have considered the exchanging antisymmetry of two identical fermions. Recall that $\phi_0(\cdot)$ is the 1D harmonic ground-state wave function and $k_z=\sqrt{2\epsilon}/d$. The two-body Green's function $G_E(\mathbf{r},\mathbf{r}')$ is the simple analytical continuation of Eq. (7) from $\epsilon<0$ to $\epsilon\approx 0^+$. Substituting the p-wave pseudopotential (3) into Eq. (25), and at large separation, i.e., $r\to\infty$, we find the scattering wave function (25) behaves as

$$\psi(\mathbf{r})\approx\phi_0\left(\frac{\sqrt{\eta}x}{d}\right)\phi_0\left(\frac{y}{d}\right)\left[i\sin(k_z z)\right.$$
$$\left.-\operatorname{sgn}(z)\left(\frac{1}{v_1}-\frac{1}{2}r_1k^2\right)^{-1}\frac{\sqrt{\pi}\eta^{1/4}\mathcal{R}_z}{d}e^{ik_z|z|}\right], \tag{27}$$

where $\mathcal{R}_z$ is defined as in Eq. (9). For this 1D scattering problem in the waveguide, we anticipate the scattering wave function at a large distance takes the form

$$\psi(\mathbf{r})\sim\phi_0\left(\frac{\sqrt{\eta}x}{d}\right)\phi_0\left(\frac{y}{d}\right)[i\sin(k_z z)+\operatorname{sgn}(z)f_pe^{ik_z|z|}], \tag{28}$$

where f_p is the effective p-wave 1D scattering amplitude. Comparing Eqs. (27) and (28), we easily obtain the effective 1D scattering amplitude

$$f_p=-\left(\frac{1}{v_1}-\frac{1}{2}r_1k^2\right)^{-1}\frac{\sqrt{\pi}\eta^{1/4}\mathcal{R}_z}{d}. \tag{29}$$

The coefficient $\mathcal{R}_z$ is determined by the asymptotic behavior of the scattering wave function (25) at a small distance, i.e., $r\approx 0$, and we find

$$\mathcal{R}_z=\frac{i6k_z\eta^{1/4}}{\sqrt{\pi}d}\left[1+\frac{6}{\sqrt{\pi}d^3}\left(\frac{1}{v_1}-\frac{1}{2}r_1k^2\right)^{-1}\mathcal{F}_z'(\epsilon,0)\right.$$
$$\left.+\frac{i6k_z\sqrt{\eta}}{d^2}\left(\frac{1}{v_1}-\frac{1}{2}r_1k^2\right)^{-1}\right]^{-1}, \tag{30}$$

where

$$\mathcal{F}_z'(\epsilon,0)=\int_0^\infty d\tau\left\{\frac{\sqrt{\eta}\exp(\epsilon\tau)}{\sqrt{2}\tau^{3/2}}\right.$$
$$\times\left[\frac{1}{\sqrt{(1-e^{-2\eta\tau})(1-e^{-2\tau})}}-1\right]$$
$$\left.-\left[\frac{1}{2\sqrt{2}\tau^{5/2}}+\frac{\epsilon+(\eta+1)/2-2\sqrt{\eta}}{2\sqrt{2}\tau^{3/2}}\right]\right\}. \tag{31}$$

For the low-energy 1D scattering problem, i.e., $k_zd\ll 1$, it is convenient to use the effective 1D scattering length l_{1D} to characterize scattering resonances, which can be defined from

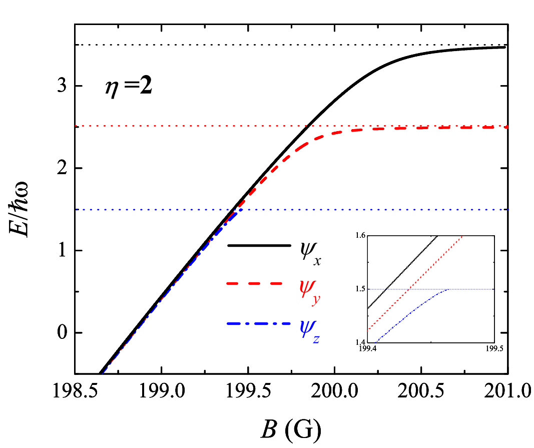

FIG. 2. (Color online) The same as Fig. 1 but for a transversely anisotropic confinement, i.e., $\eta=2$.

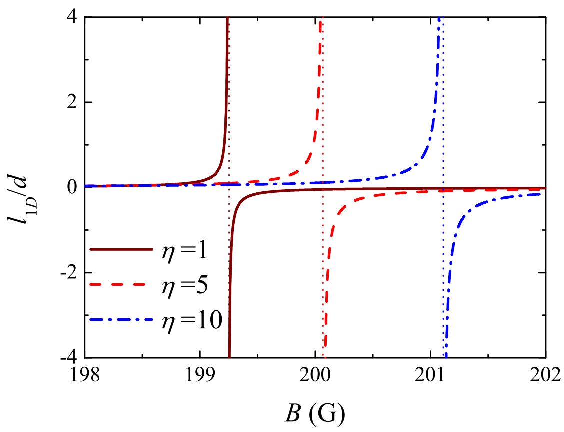

FIG. 3. (Color online) The p-wave 1D scattering length l_{1D} as a function of the magnetic field B in the 1D waveguide with different transverse anisotropy η. Here, the p-wave interaction of two ^{40}K atoms in the $|F = \frac{9}{2}, m_F = -\frac{7}{2}\rangle$ hyperfine state is tuned by using the p-wave Feshbach resonance centered at $B_0 = 198.8$ G in three dimensions [7,24].

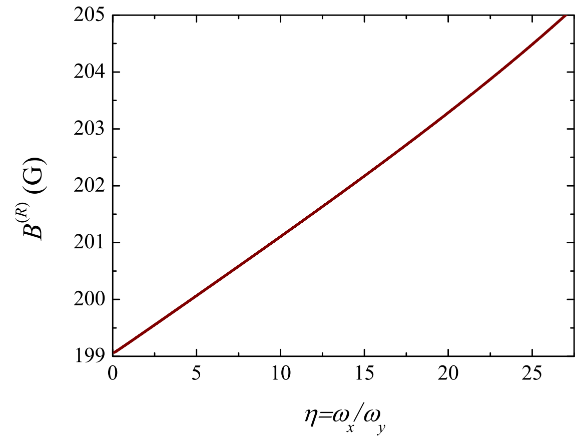

FIG. 4. (Color online) The position of the p-wave confinement-induced resonance in the 1D waveguide denoted by the magnetic field strength $B^{(R)}$ as a function of the transverse anisotropy η. The transverse anisotropy η can be tuned by increasing ω_x with a fixed ω_y. The corresponding parameters are from the ^{40}K experiments [7,24].

the scattering amplitude at zero energy ($\epsilon \approx 0^+$) as [8]

$$f_p = -\frac{ik_z}{ik_z + l_{1D}^{-1}}, \tag{32}$$

and

$$\frac{l_{1D}}{d} \equiv 6\sqrt{\eta}\left[\frac{d^3}{v_1} - dr_1\frac{\eta+1}{2} + \frac{6}{\sqrt{\pi}}\mathcal{F}_z'(0,0)\right]^{-1}. \tag{33}$$

The divergence of the 1D scattering length l_{1D} characterizes the scattering resonance in the waveguide. By tuning the 3D scattering volume v_1 and the effective range r_1 according to the p-wave Feshbach resonance, the p-wave CIR can be reached in the 1D waveguide when

$$\frac{d^3}{v_1} - dr_1\frac{\eta+1}{2} = -\frac{6}{\sqrt{\pi}}\mathcal{F}_z'(0,0). \tag{34}$$

For a transversely isotropic confinement, i.e., $\eta = 1$, the 1D resonance condition (34) returns to the same result with [5,8]

$$\frac{d^3}{v_1} - dr_1 = 12\zeta\left(-\frac{1}{2}\right), \tag{35}$$

where $\zeta(\cdot)$ is the Riemann zeta function, and we note that the 3D p-wave effective range r_1 is included.

In Fig. 3, we present the 1D scattering length l_{1D} as a function of the magnetic field strength B in the waveguide with three typical transverse anisotropies $\eta = 1$, 5, and 10. Here, we still consider two ^{40}K atoms in the $|F = \frac{9}{2}, m_F = -\frac{7}{2}\rangle$ hyperfine state near the p-wave Feshbach resonance centered at $B_0 = 198.8$ G in three dimensions [7,24]. As the transverse anisotropy η increases, the resonance position of p-wave CIR shifts to a higher magnetic field strength. More apparently, we show how the resonance position denoted by the magnetic field strength $B^{(R)}$ shifts with the transverse anisotropy η in Fig. 4.

IV. THE EFFECTIVE TWO-CHANNEL MECHANISM

For the s-wave CIR in a quasi-1D waveguide, there is a simple two-channel picture first introduced by Bergeman *et al.* [4], which is then extended to the case under a transversely anisotropic confinement [12]. We may understand this two-channel mechanism as follows: due to the low temperature and tight transverse confinement, the two atoms can only enter from the transverse ground state, while the transverse excited modes are all closed. Therefore, the transverse ground state and the manifold of the remaining transverse excited states may be regarded as the open and closed channels, respectively. If a molecular state exists in the closed channel, a zero-energy scattering resonance occurs when this molecule energetically coincides with the continuum threshold of the open channel. In this section, we aim to study whether this simple two-channel picture is still valid for the p-wave interaction.

Owing to the separability of c.m. motion and relative motion in harmonic traps, we still only focus on the relative-motion Hamiltonian (1). According to the two-channel mechanism proposed for the s-wave interaction [4], the total relative-motion Hamiltonian $\mathcal{H}$ may formally be split into three terms, i.e., $\mathcal{H}_{op}$, $\mathcal{H}_{cl}$, and $\mathcal{W}$ corresponding to the open channel, the closed channel, and the coupling part, respectively,

$$\begin{aligned}\mathcal{H} &= \mathcal{H}_{op} + \mathcal{H}_{cl} + \mathcal{W}\\ &\equiv P_g\mathcal{H}P_g + P_e\mathcal{H}P_e + (P_g\mathcal{H}P_e + \text{H.c.}),\end{aligned} \tag{36}$$

where $P_g = |g\rangle\langle g|$, $P_e = \sum_{\alpha\neq g}|\alpha\rangle\langle\alpha|$ are the corresponding projection operators, and $|g\rangle$, $|\alpha\rangle$ are the transverse ground and excited states, respectively. The crossing of the molecular state in the closed channel $\mathcal{H}_{cl}$ and the energy continuum threshold of the open channel, i.e., $\hbar\omega(\eta+1)/2$, denotes the position where the p-wave CIR occurs.

In order to obtain the molecular state in the closed channel, we solve the Schrödinger equation $\mathcal{H}_{cl}\psi_{cl}(\mathbf{r}) = E\psi_{cl}(\mathbf{r})$ with a procedure similar to that used to obtain the bound states of the full Hamiltonian (1). However, the transverse ground state should be projected out, which means the transverse ground state, i.e., the $n_1 = n_2 = 0$ term, should be excluded in the expansion of the Green's function, i.e., Eq. (6). We obtain the Green's function for the closed-channel Hamiltonian,

$$G_E^{(cl)}(\mathbf{r},\mathbf{r}') = \frac{\sqrt{\eta}}{\pi^{3/2}d^3\hbar\omega}\exp\left[-\frac{\eta(x^2+x'^2)+y^2+y'^2}{2d^2}\right]\int_0^\infty d\tau\frac{e^{\epsilon\tau}}{\sqrt{2\tau}}\exp\left[-\frac{(z-z')^2}{2d^2\tau}\right] \times\left\{\frac{\exp\left[\eta\frac{2xx'-(x'^2+x^2)e^{-\eta\tau}}{2d^2\sinh(\eta\tau)}+\frac{2yy'-(y'^2+y^2)e^{-\tau}}{2d^2\sinh\tau}\right]}{\sqrt{(1-e^{-2\eta\tau})(1-e^{-2\tau})}}-1\right\}. \quad (37)$$

Then the bound-state wave function $\psi_{cl}(\mathbf{r})$ takes the form

$$\psi_{cl}(\mathbf{r}) = -\frac{\pi\hbar^2}{\mu}\left(\frac{1}{v_1}-\frac{1}{2}r_1k^2\right)^{-1} \times\sum_n\mathcal{R}'_n\left[\frac{\partial}{\partial r'_n}G_E^{(cl)}(\mathbf{r},\mathbf{r}')\right]_{r'=0}, \quad (38)$$

where

$$\mathcal{R}'_n \equiv \left[\frac{\partial^3}{\partial r^3}r^3\frac{\partial}{\partial r_n}\psi_{cl}(\mathbf{r})\right]. \quad (39)$$

After straightforward and similar algebra as that in Sec. II, we find there are also three bound states in the closed channel. Two of them have odd transverse parity, while the third is even, which takes the form

$$\psi_{cl,z}(\mathbf{r}) = z\left[\frac{1}{r^3}+\frac{E}{\hbar\omega}\frac{1}{rd^2}+\frac{2}{\sqrt{\pi}d^3}\mathcal{F}'_z(\epsilon,\mathbf{r})\right], \quad (40)$$

where

$$\mathcal{F}'_z(\epsilon,\mathbf{r}) = \int_0^\infty d\tau\left\{\frac{\sqrt{\eta}\exp\left(\epsilon\tau-\frac{z^2}{2d^2\tau}-\frac{\eta x^2+y^2}{2d^2}\right)}{\sqrt{2}\tau^{3/2}} \times\left[\frac{\exp\left(-\frac{\eta x^2e^{-\eta\tau}}{2d^2\sinh(\eta\tau)}-\frac{y^2e^{-\tau}}{2d^2\sinh\tau}\right)}{\sqrt{(1-e^{-2\eta\tau})(1-e^{-2\tau})}}-1\right] -\frac{1}{2\sqrt{2}}\left[\frac{1}{\tau^{5/2}}+\left(\frac{E}{\hbar\omega}-2\sqrt{\eta}\right)\frac{1}{\tau^{3/2}}\right]e^{-\frac{r^2}{2d^2\tau}}\right\}. \quad (41)$$

The binding energy of the bound state with even transverse parity satisfies

$$\frac{d^3}{v_1} = -\frac{6}{\sqrt{\pi}}\mathcal{F}'_z(\epsilon,0)+dr_1\frac{E}{\hbar\omega}. \quad (42)$$

Obviously, $\mathcal{F}'_z(\epsilon,0)$ takes the form of Eq. (31). If two atoms enter from the transverse ground state (open channel) with energy below the transverse excited states (closed channel), they may only be coupled to the molecular state $\psi_{cl,z}(\mathbf{r})$

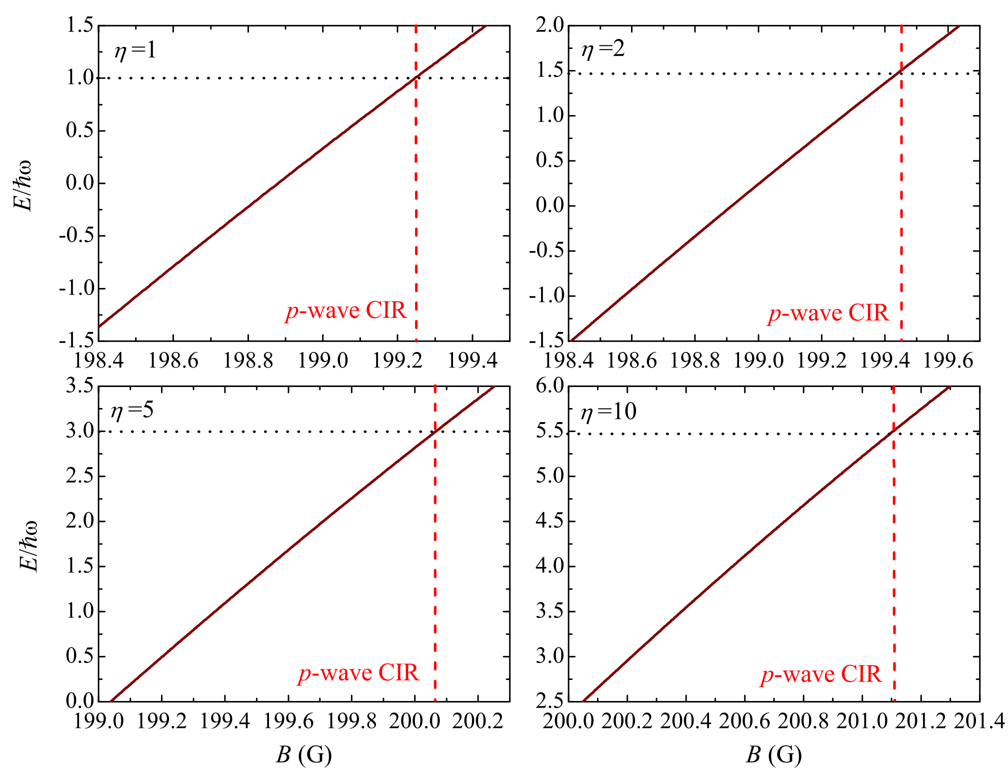

FIG. 5. (Color online) The molecular state in the closed-channel Hamiltonian $\mathcal{H}_{cl}$ for different transverse anisotropy η. The horizontal dotted lines denote the continuum threshold of the open channel, while the vertical red dashed lines indicate the position where a zero-energy scattering resonance occurs.

with even transverse parity, due to the parity conservation. When this molecular state energetically crosses the continuum threshold of the open channel ($\epsilon \approx 0$), a zero-energy scattering resonance occurs. From Eq. (40), the resonance condition is given by

$$\frac{d^3}{v_1} - dr_1 \frac{\eta + 1}{2} = -\frac{6}{\sqrt{\pi}} \mathcal{F}_z'(0,0), \tag{43}$$

which returns to Eq. (34) again. The binding energy of the ^{40}K-^{40}K molecular state $\psi_{cl,z}(\mathbf{r})$ in the closed-channel Hamiltonian $\mathcal{H}_{cl}$ under different transverse anisotropy is illustrated in Fig. 5, where the atomic interaction is still tuned by the p-wave Feshbach resonance centered at $B_0 = 198.8$ G. The vertical dashed lines indicate the position where the p-wave CIR occurs.

V. CONCLUSIONS

We theoretically investigate the influence of the transverse anisotropy of confinement on the two-body state with p-wave interaction in a quasi-one-dimensional waveguide, which is a useful generalization of Pricoupenko's theory. The two-body problem in such quasi-one-dimensional systems is solved, where the interatomic interaction is modeled by the p-wave pseudopotential. There are a total of three bound states due to the nonzero orbital angular momentum of the p-wave interaction, while there is only one two-body bound state supported by s-wave pseudopotential. In addition, the effective one-dimensional scattering amplitude and scattering length are derived. We predict the p-wave confinement-induced resonance for different transverse anisotropy of the waveguide, whose position shifts with increasing transverse anisotropy. In addition, we find that an effective two-channel mechanism is still valid for the p-wave confinement-induced resonance in a waveguide, which was first proposed for the s-wave interaction. All our calculations are based on the parametrization of the ^{40}K experiments and thus can be confirmed in future experiments.

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