Manipulation of p-Wave Scattering of Cold Atoms in Low Dimensions Using the Magnetic Field Vector

Shi-Guo Peng,[1] Shina Tan,[2,3,*] and Kaijun Jiang[1,3,†]
[1]State Key Laboratory of Magnetic Resonance and Atomic and Molecular Physics, Wuhan Institute of Physics and Mathematics, Chinese Academy of Sciences, Wuhan 430071, China
[2]School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332, USA
[3]Center for Cold Atom Physics, Chinese Academy of Sciences, Wuhan 430071, China
(Received 11 December 2013; published 23 June 2014)

It is well known that the magnetic Feshbach resonances of cold atoms are sensitive to the magnitude of the external magnetic field. Much less attention has been paid to the direction of such a field. In this work we calculate the scattering properties of spin polarized fermionic atoms in reduced dimensions, near a p-wave Feshbach resonance. Because of the spatial anisotropy of the p-wave interaction, the scattering has a nontrivial dependence on both the magnitude and the direction of the magnetic field. In addition, we identify an inelastic scattering process which is impossible in the isotropic-interaction model; the rate of this process depends considerably on the direction of the magnetic field. Significantly, an Einstein-Podolsky-Rosen entangled pair of identical fermions may be produced during this inelastic collision. This work opens a new method to manipulate resonant cold atomic interactions.

DOI: 10.1103/PhysRevLett.112.250401 PACS numbers: 03.75.Ss, 05.30.Fk, 34.50.−s, 67.85.−d

Introduction.—Unlike electrons in condensed matter and nucleons inside a nucleus, ultracold atoms have tunable interactions thanks to the magnetic Feshbach resonances [1]. The direction of the magnetic field plays little role in isotropic s-wave interactions, other than providing a quantization axis for the hyperfine states. For p-wave interactions, however, the resonance positions for different orbital magnetic quantum numbers may be different due to the anisotropic magnetic dipole-dipole interaction [2,3]. The physical implication of this split for cold atomic scatterings in reduced dimensions has not been explored theoretically. In previous theoretical work on these scatterings, such split was not taken into account [4,5]. While this omission is reasonable for atoms with small magnetic dipoles such as ^{6}Li [6], we have to consider the effect of the split for other atoms such as ^{40}K which show large splits [2,3].

In reduced dimensions, new two-body scattering resonances known as confinement-induced resonances (CIRs) [7] appear. Recently an impressive amount of work was devoted to the s-wave CIRs [8–11].

In this Letter we study the scattering of two spin-polarized fermionic atoms near p-wave Feshbach resonances, confined in low dimensions. We will assume that the oscillator length in the confinement directions is much larger than the tiny van der Waals length scale. Because of the split of resonance positions for different orbital magnetic quantum numbers, we find that (i) one can tune the scattering properties continuously by changing the direction of the magnetic field, (ii) a new inelastic scattering process where one of the two atoms is excited in the confined direction by $\hbar\omega$ is now possible, where ω is the angular frequency for the confinement, and (iii) in quasi-one-dimensional (quasi-1D), by choosing the magnitude and direction of the field and the collision energy judiciously, one can have 100% probability for this inelastic process, thus creating an Einstein-Podolsky-Rosen (EPR) entangled pair.

Short-range boundary conditions.—In the presence of a uniform external magnetic field $\mathbf{B}$, if two fermionic atoms in the same hyperfine state have a resonant p-wave interaction but negligible interactions in higher partial waves, their relative wave function has a partial wave expansion,

$$\psi(\mathbf{r}) = \sum_{m=-1}^{1}\left[\frac{\mathcal{A}_m}{r^2} + \mathcal{B}_m + \mathcal{C}_m r + O(r^2)\right] Y_{1m}(\hat{\mathbf{r}}) + O(r^3), \tag{1}$$

at a distance r that is small compared to the average interatomic distance but still large compared to the range of interaction. Here $Y_{1m}(\hat{\mathbf{r}})$ are spherical harmonics whose north pole is in the direction of $\mathbf{B}$.

We assume that the scattering phase shifts δ_{1m} for different m's may be different; at small collision energies we shall take the following effective range expansion [2]:

$$k^3 \cot\delta_{1m} = -v_{1m}^{-1} + r_{1m}k^2/2, \tag{2}$$

where v_{1m} and r_{1m} are respectively the p-wave scattering volume and effective range. Unlike in the s-wave interaction, the p-wave effective range is essential near resonance [12–14]. From Eq. (2) we can easily derive three linear constraints on the coefficients $\mathcal{A}_m$, $\mathcal{B}_m$, and $\mathcal{C}_m$,

$$v_{1m}^{-1}\mathcal{A}_m - r_{1m}\mathcal{B}_m + 3\mathcal{C}_m = 0, \qquad (m = 0, \pm 1), \quad (3)$$

which generalize the boundary condition in Ref. [15] to anisotropic p-wave interactions. Unlike Eq. (2), Eq. (3) is also applicable to the quantum states without a certain collision energy. In this sense Eq. (3) is similar to the Bethe-Peierls boundary condition [16] for s-wave interactions. We expect Eq. (3) will be very useful in the theory of N cold atoms with anisotropic p-wave interactions.

Quasi-2D Fermi gases.—Consider the scattering of two fermionic atoms in the same hyperfine state, subject to a tight harmonic confinement along the z axis. The wave function $\psi(\mathbf{r})$ for the relative motion satisfies the Schrödinger equation $(-\hbar^2\nabla^2/2\mu + \mu\omega^2 z^2/2)\psi = E\psi$ outside the range of interaction, where μ is the reduced mass, and E is the relative collision energy which excludes the center-of-mass energy. Without loss of generality we assume the external magnetic field lies in the xz plane, and forms an angle α with the $+z$ axis. Assuming that the incoming atoms are in the axial ground state, and the incident direction forms an angle β with the $+x$ axis, we obtain the scattering wave function

$$\psi(\mathbf{r}) = \sin[q_0\rho\cos(\varphi - \beta)]e^{-(z^2/2d^2)} + \sum_n W_n r_n \mathcal{L}_n(\epsilon, \mathbf{r}), \quad (4)$$

where $\epsilon \equiv \frac{E}{\hbar\omega} - \frac{1}{2}$ for this quasi-2D geometry, ρ and φ are polar coordinates: $\rho\cos\varphi = x$, $\rho\sin\varphi = y$, $d = \sqrt{\hbar/\mu\omega}$ is the oscillator length, $q_0 = \sqrt{2\epsilon}/d$, the summation is over $n = x, y, z$ with $r_x \equiv x$, $r_y \equiv y$, $r_z \equiv z$, and the functions $\mathcal{L}_n(\epsilon, \mathbf{r})$ are related to the gradients of the Green's function of the 3D harmonic oscillator Hamiltonian (see Appendix E of Ref. [17]),

$$\mathcal{L}_n(\epsilon, \mathbf{r}) = \frac{1}{\sqrt{2\pi}d^3}\int_0^\infty d\tau \frac{e^{(\epsilon+(1/2))\tau-(\rho^2/2d^2\tau)-(z^2/2d^2\tanh\tau)}}{\tau^{2-j_n}\sinh^{j_n+(1/2)}\tau}, \quad (5)$$

for $\epsilon < j_n$. Here $j_x = j_y = 0$ and $j_z = 1$. To have outgoing scattered waves at higher energies, we should analytically continue $\mathcal{L}_n(\epsilon, \mathbf{r})$ from $\epsilon < j_n$ to $\epsilon > j_n$ along routes *above* the real-ϵ axis.

To determine the coefficients W_n in Eq. (4), we make a coordinate transformation $x = x'\cos\alpha + z'\sin\alpha$, $y = y'$, $z = -x'\sin\alpha + z'\cos\alpha$. The z' axis is parallel to the magnetic field. Expanding ψ at small (x', y', z') and matching the boundary conditions Eq. (3), we find

$$W_x = -\frac{3d^3 q_0(D_{0z}\cos^2\alpha + D_{1z}\sin^2\alpha)\cos\beta}{D_{0z}D_{1x}\cos^2\alpha + D_{0x}D_{1z}\sin^2\alpha}, \quad (6a)$$

$$W_y = -\frac{3d^3 q_0 \sin\beta}{D_{1x}}, \quad (6b)$$

$$W_z = \frac{3d^3 q_0(D_0 - D_1)\cos\beta\sin\alpha\cos\alpha}{D_{0z}D_{1x}\cos^2\alpha + D_{0x}D_{1z}\sin^2\alpha}, \quad (6c)$$

where $D_{mn} = D_m + c_n(\epsilon)$, $D_m = v_{1m}^{-1}d^3 - (\epsilon + (1/2))r_{1m}d$, and $c_n(\epsilon) = \lim_{r\to 0} 3d^3[\mathcal{L}_n(\epsilon, \mathbf{r}) - r^{-3} - (\epsilon + (1/2))/d^2 r]$. Note that $c_n(\epsilon)$ are pure mathematical functions [18].

If $0 < \epsilon < 1$, both atoms will remain in the axial ground state after the collision; i.e. the collision is purely elastic. At large ρ the wave function takes the form

$$\psi(\mathbf{r}) \approx -\frac{i}{2}e^{-(z^2/2d^2)}\left\{2i\sin[q_0\rho\cos(\varphi-\beta)] + \frac{f_{00}(\varphi)}{\sqrt{\rho}}e^{iq_0\rho}\right\}, \quad (7)$$

where $f_{00}(\varphi)$ is the elastic scattering amplitude,

$$f_{00}(\varphi) = e^{\pi i/4}(2\sqrt{2q_0}/d)(W_x\cos\varphi + W_y\sin\varphi). \quad (8)$$

Doing a partial-wave expansion at large ρ, we find ψ is a linear combination of $e^{-z^2/2d^2}\rho^{-1/2}\cos(q_0\rho + \frac{\pi}{4} + \delta_x)\cos\varphi$ and $e^{-z^2/2d^2}\rho^{-1/2}\cos(q_0\rho + \frac{\pi}{4} + \delta_y)\sin\varphi$ plus higher partial wave components. The phase shifts δ_x and δ_y are independent of the incident angle β,

$$e^{2i\delta_x} = 1 + \frac{i\sqrt{\pi}q_0}{d}\frac{W_x}{\cos\beta}; \qquad e^{2i\delta_y} = 1 + \frac{i\sqrt{\pi}q_0}{d}\frac{W_y}{\sin\beta}. \quad (9)$$

We find the energy-dependent scattering areas for the p-wave scattering in two dimensions,

$$s_x = -\frac{\tan\delta_x}{q_0^2} = \frac{3\sqrt{\pi}d^2(D_{0z}\cos^2\alpha + D_{1z}\sin^2\alpha)}{2\mathrm{Re}(D_{0z}D_{1x}\cos^2\alpha + D_{0x}D_{1z}\sin^2\alpha)}, \quad (10a)$$

$$s_y = -\frac{\tan\delta_y}{q_0^2} = \frac{3\sqrt{\pi}d^2}{2\mathrm{Re}D_{1x}}, \quad (10b)$$

where Re stands for the real part. The total elastic scattering cross section $\sigma_{00} = \frac{1}{2}\int_0^{2\pi}|f_{00}(\varphi)|^2 d\varphi$ yields

$$\sigma_{00} = \frac{16}{q_0}(\sin^2\delta_x\cos^2\beta + \sin^2\delta_y\sin^2\beta). \quad (11)$$

If $1 < \epsilon < 2$, one atom may go to the axial first excited state after the collision. At large ρ we have approximately

$$\psi(\mathbf{r}) \propto \{2i\sin[q_0\rho\cos(\varphi - \beta)] + f_{00}(\varphi)\rho^{-1/2}e^{iq_0\rho}\}\phi_0(z) + f_{01}\rho^{-1/2}e^{iq_1\rho}\phi_1(z), \quad (12)$$

where $q_1 = \sqrt{2(\epsilon - 1)}/d$, $\phi_0(z) = (1/\pi^{1/4}d^{1/2})e^{-z^2/2d^2}$ and $\phi_1(z) = (\sqrt{2}/\pi^{1/4}d^{3/2})ze^{-z^2/2d^2}$ are normalized axial energy eigenfunctions, $f_{00}(\varphi)$ is still given by Eq. (8), and $f_{01} = 4e^{3\pi i/4}W_z/(d^2\sqrt{q_1})$. The total scattering cross section is now $\sigma = \sigma_{00} + \sigma_{01}$, with σ_{00} being the elastic cross section [still given by Eq. (11)], and $\sigma_{01} = (q_1/2q_0)\int_0^{2\pi}|f_{01}|^2 d\varphi$ the inelastic cross section. We find

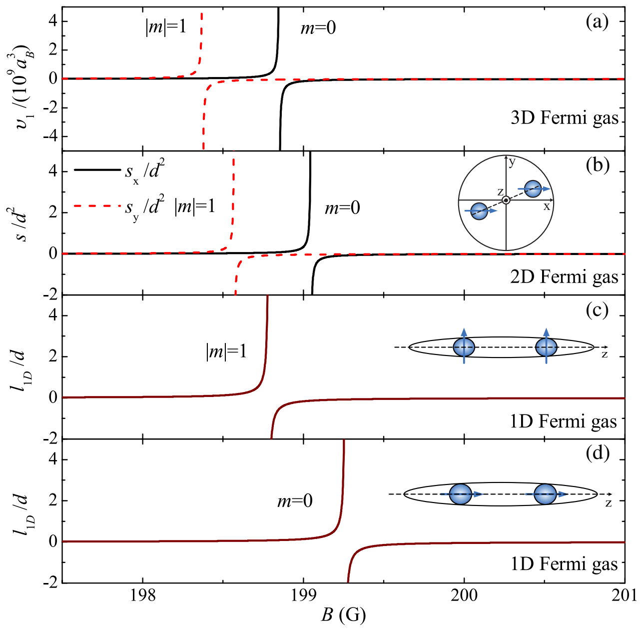

FIG. 1 (color online). Resonant scattering properties of two spin-polarized ^{40}K fermionic atoms in the $|F=9/2, m_F=-7/2\rangle$ hyperfine state as functions of the external magnetic field strength. (a) The p-wave scattering volume v_{1m} in three dimensions [2]. (b) The 2D scattering areas s_x and s_y in quasi-2D at the scattering threshold ($\epsilon=0$), with the magnetic field being parallel to the plane wherein atoms lie (i.e., $\alpha=\pi/2$). (c) The 1D scattering length l_{1D} in quasi-1D at the scattering threshold ($\epsilon=0$), with the magnetic field being perpendicular to the waveguide (i.e., $\alpha=\pi/2$), and (d) the magnetic field is parallel to the waveguide (i.e., $\alpha=0$). Here a_B is the Bohr radius. To compare with the experiments, we have chosen the corresponding parameters from Ref. [3], and the effective-range-expansion parametrization from Ref. [2].

$$\sigma_{01}=\frac{144\pi\sqrt{2\epsilon}d(D_0-D_1)^2\cos^2\beta\sin^2\alpha\cos^2\alpha}{|D_{0z}D_{1x}\cos^2\alpha+D_{0x}D_{1z}\sin^2\alpha|^2}. \qquad (13)$$

The occurrence of this inelastic process has the following three necessary conditions: (i) the phase shifts for $m=0$ and $|m|=1$ should be different, so that $D_0\neq D_1$, (ii) the magnetic field is neither parallel nor perpendicular to the quasi-2D plane, i.e., $\alpha\neq 0, \pi/2$, and (iii) the magnetic field is not perpendicular to the incident direction, so that $\beta\neq\pi/2$.

As a consistency check, one can verify that the total cross section obeys the optical theorem in 2D [19]: $\sigma=2\sqrt{2\pi/q_0}\mathrm{Re}[e^{-3\pi i/4}f_{00}(\beta)]$.

Let us illustrate our results using the parameters in the quasi-2D ^{40}K experiment [3]. In Fig. 1(b) we plot the p-wave scattering areas versus the magnetic field strength at $\epsilon=0$. The resonance positions are shifted from their free-space values, mainly due to the zero-point energy, and they agree with Ref. [3]. In Fig. 2 (left column) we only show how the resonances in s_x change as the magnetic field is tilted, since s_y does not depend on α [s_y is shown in Fig. 1(b)]. If $\alpha=0$, there is only one resonance in s_x, corresponding to the $|m|=1$ channels. As α increases, one additional resonance in s_x corresponding to the $m=0$ channel appears. The relative widths of the two resonances change, and their positions shift slightly as well. When $\alpha=\pi/2$, the $|m|=1$ resonance in s_x disappears, while the $|m|=1$ resonance in s_y still exists [see Fig. 1(b)]. In Fig. 3 (top) we show the anisotropy of the p-wave interaction by plotting the elastic differential cross section $|f_{00}(\varphi)|^2$ at $\epsilon=0.01$ and $B=198.574$ G, which is detuned by approximately -0.001 G from the free space $|m|=1$ resonance at collision energy $0.51\hbar\omega$. In Fig. 3 (bottom) we plot the

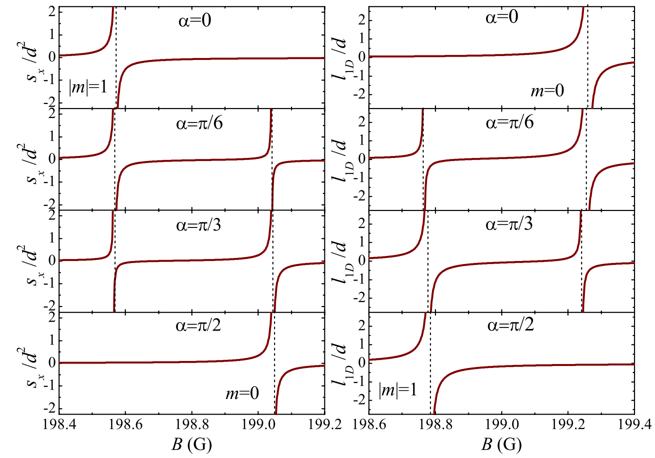

FIG. 2 (color online). Left column: the 2D scattering area s_x at $\epsilon=0$ for various angles between the field and the normal direction of the quasi-2D plane. Right column: the 1D scattering length l_{1D} at $\epsilon=0$ for various angles between the field and the quasi-1D line. Dashed lines indicate the locations of resonances. Parameters are the same as in Fig. 1.

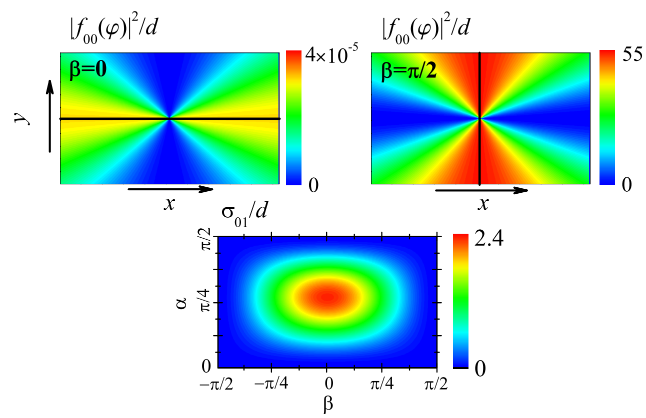

FIG. 3 (color online). Top: the angular distributions of the elastic differential cross section $|f_{00}(\varphi)|^2$ in quasi-2D for different incident directions (black lines) at $\epsilon=0.01$, $\alpha=\pi/2$, and $B=198.574$ G. The corresponding scattering phase shifts are $\delta_x\approx-0.037°$ and $\delta_y\approx-59.588°$. Bottom: the inelastic cross section σ_{01} varying with the relative direction of the magnetic field at $\epsilon=1.5$ and $B=199.623$ G. All other parameters are the same as in Fig. 1.

inelastic cross section σ_{01} at $\epsilon = 1.5$ and $B = 199.623$ G (detuned by approximately -0.014 G from the free space $m = 0$ resonance at collision energy $2\hbar\omega$). We find that σ_{01} has a broad peak as a function of the direction of the field relative to the incident direction of the two atoms.

Quasi-1D Fermi gases.—We now consider the scattering of two fermionic atoms in the same hyperfine state, subject to an axially symmetric harmonic confinement toward the z axis. The wave function $\psi_{1\mathrm{D}}(\mathbf{r})$ for the relative motion satisfies the Schrödinger equation $(-\hbar^2\nabla^2/2\mu + \mu\omega^2\rho^2/2)\psi_{1\mathrm{D}} = E\psi_{1\mathrm{D}}$ outside the range of interaction, where μ is the reduced mass, $\rho = \sqrt{x^2+y^2}$, and E is the relative energy which excludes the center-of-mass energy. Let the external magnetic field lie in the xz plane, and form an angle α with the $+z$ axis. Assuming that the incoming atoms are in the transverse ground state, we obtain the scattering wave function

$$\psi_{1\mathrm{D}}(\mathbf{r}) = e^{-\rho^2/2d^2}\sin(q_0 z) + \sum_n W_n^{1\mathrm{D}} r_n \mathcal{L}_n^{1\mathrm{D}}(\epsilon,\mathbf{r}), \quad (14)$$

where $\epsilon \equiv (E/\hbar\omega) - 1$ for this quasi-1D geometry, $d = \sqrt{\hbar/\mu\omega}$, $q_0 = \sqrt{2\epsilon}/d$, the summation is over $n = x, y, z$, and

$$\mathcal{L}_n^{1\mathrm{D}}(\epsilon,\mathbf{r}) = \frac{1}{\sqrt{2\pi}d^3}\int_0^\infty d\tau \frac{e^{(\epsilon+1)\tau-(\rho^2/2d^2\tanh\tau)-(z^2/2d^2\tau)}}{\tau^{(3/2)-j_n}\sinh^{j_n+1}\tau} \quad (15)$$

for $\epsilon < j_n$. Here $j_x = j_y = 1$ and $j_z = 0$. To have outgoing scattered waves at higher energies, we analytically continue $\mathcal{L}_n^{1\mathrm{D}}(\epsilon,\mathbf{r})$ toward greater values of ϵ along routes *above* the real-ϵ axis. Taking the same coordinate transformation as in the quasi-2D case, and using the boundary conditions Eq. (3), we find

$$W_x^{1\mathrm{D}} = \frac{3d^3 q_0 (D_0^{1\mathrm{D}} - D_1^{1\mathrm{D}})\sin\alpha\cos\alpha}{D_{0z}^{1\mathrm{D}} D_{1x}^{1\mathrm{D}}\cos^2\alpha + D_{0x}^{1\mathrm{D}} D_{1z}^{1\mathrm{D}}\sin^2\alpha}, \quad (16a)$$

$$W_y^{1\mathrm{D}} = 0, \quad (16b)$$

$$W_z^{1\mathrm{D}} = -\frac{3d^3 q_0 (D_{1x}^{1\mathrm{D}}\cos^2\alpha + D_{0x}^{1\mathrm{D}}\sin^2\alpha)}{D_{0z}^{1\mathrm{D}} D_{1x}^{1\mathrm{D}}\cos^2\alpha + D_{0x}^{1\mathrm{D}} D_{1z}^{1\mathrm{D}}\sin^2\alpha}, \quad (16c)$$

where $D_{mn}^{1\mathrm{D}} = D_m^{1\mathrm{D}} + c_n^{1\mathrm{D}}(\epsilon)$, $D_m^{1\mathrm{D}} = v_{1m}^{-1}d^3 - (\epsilon+1)r_{1m}d$, and $c_n^{1\mathrm{D}}(\epsilon) = \lim_{r\to 0} 3d^3[\mathcal{L}_n^{1\mathrm{D}}(\epsilon,\mathbf{r}) - r^{-3} - (\epsilon+1)/d^2 r]$. Note that $c_n^{1\mathrm{D}}(\epsilon)$ are pure mathematical functions [20].

For $0 < \epsilon < 2$, at $|z| \to \infty$ we find asymptotically

$$\psi_{1\mathrm{D}}(\mathbf{r}) \propto \mathrm{sgn}(z)(e^{-iq_0|z|} + g_{00}e^{iq_0|z|})e^{-\rho^2/2d^2} + \theta(\epsilon-1)g_{01}e^{iq_1|z|}(\sqrt{2}x/d)e^{-\rho^2/2d^2}, \quad (17)$$

where $\mathrm{sgn}(z)$ is the sign of z, θ is the unit step function, $q_1 = \sqrt{2(\epsilon-1)}/d$, $g_{00} = -1 - 4iW_z^{1\mathrm{D}}/d^2$, and $g_{01} = 4W_x^{1\mathrm{D}}/(\sqrt{\epsilon-1}d^2)$.

In the window of energies $0 < \epsilon < 1$, the 1D scattering is purely elastic, with a 1D phase shift $\delta_{1\mathrm{D}}$ given by the equation $g_{00} = -e^{2i\delta_{1\mathrm{D}}}$. The energy-dependent 1D scattering length is

$$l_{1\mathrm{D}} = -\frac{\tan\delta_{1\mathrm{D}}}{q_0} = \frac{6d(D_{1x}^{1\mathrm{D}}\cos^2\alpha + D_{0x}^{1\mathrm{D}}\sin^2\alpha)}{\bar{D}_{0z}^{1\mathrm{D}} D_{1x}^{1\mathrm{D}}\cos^2\alpha + D_{0x}^{1\mathrm{D}}\bar{D}_{1z}^{1\mathrm{D}}\sin^2\alpha}, \quad (18)$$

where $\bar{D}_{mz}^{1\mathrm{D}} \equiv D_{mz}^{1\mathrm{D}} - i6\sqrt{2\epsilon} = D_m^{1\mathrm{D}} - 12\zeta(-(1/2),(2-\epsilon/2))$, and $\zeta(s,a)$ is the Hurwitz zeta function [21].

For isotropic interactions, $D_0^{1\mathrm{D}} = D_1^{1\mathrm{D}}$, our result reduces to $l_{1\mathrm{D}} = 6d/\bar{D}_{mz}^{1\mathrm{D}}$, which agrees with Ref. [4] but disagrees somewhat with Ref. [5]. At $\epsilon = 0$ we find $l_{1\mathrm{D}} = 6d/[(d/a_1)^3 - r_1 d - 12\zeta(-(1/2))]$, where the denominator contains an extra term $-r_1 d$ compared to Eq. (9) of Ref. [5], because of a finite zero-point energy $\hbar\omega$ for the collision. For anisotropic interactions, $D_0^{1\mathrm{D}} \neq D_1^{1\mathrm{D}}$, we should use the more general formula Eq. (18), and $l_{1\mathrm{D}}$ depends on the direction of the magnetic field. In Fig. 1(c),(d) we illustrate this by plotting $l_{1\mathrm{D}}$ for ^{40}K at $\epsilon = 0$ for two different directions of the field. The atom loss peaks observed in Ref. [3] were about $0.2 \sim 0.3$ G to the right of the resonance positions shown in Figs. 1(c) and 1(d), probably because of the finite Fermi energy and thermal effects in the experiment. In Fig. 2 (right column) we show how the resonances in $l_{1\mathrm{D}}$ change as the magnetic field is tilted. The physics is somewhat analogous to the 2D case, but we see more appreciable shifts of the resonance positions as α changes.

In the window of energies $1 < \epsilon < 2$, one atom may go to a transverse first excited state after the collision. The transmission coefficient to this excited state is

$$\mathcal{T} = \frac{q_1}{q_0}|g_{01}|^2 = \frac{288\sqrt{\frac{\epsilon}{\epsilon-1}}(D_0^{1\mathrm{D}} - D_1^{1\mathrm{D}})^2\sin^2\alpha\cos^2\alpha}{|D_{0z}^{1\mathrm{D}} D_{1x}^{1\mathrm{D}}\cos^2\alpha + D_{0x}^{1\mathrm{D}} D_{1z}^{1\mathrm{D}}\sin^2\alpha|^2}. \quad (19)$$

The probability of the elastic scattering is $\mathcal{R} = |g_{00}|^2$. For $1 < \epsilon < 2$ one can verify that $\mathcal{R} + \mathcal{T} = 1$.

In Fig. 4 we plot the variation of the inelastic transmission coefficient with the direction of the magnetic field at a magic value of magnetic field strength, at which the peak value of $\mathcal{T}$ is 1. At each collision energy satisfying $\epsilon_c < \epsilon < 2$, we find two magic points on the parameter plane (B, α), such that $g_{00} = 0$ and the transmission coefficient $\mathcal{T}$ reaches unity [22]. When this happens, one and only one atom is excited transversely, but we do not know which, and the outgoing state is an EPR pair with maximum entanglement, with two-body wave function

$$\Psi \propto e^{iq_1|z_1-z_2|}[\phi_0(x_1,y_1)\phi_x(x_2,y_2) - \phi_x(x_1,y_1)\phi_0(x_2,y_2)], \quad (20)$$

where (x_i, y_i, z_i) are the coordinates of the ith atom, ϕ_0 is the transverse ground state, and $\phi_x(x,y) \propto x\phi_0(x,y)$ is a transverse excited state wave function. The two magic points

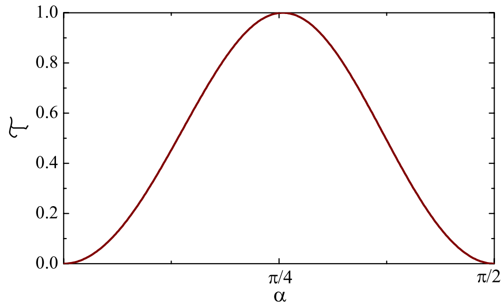

FIG. 4 (color online). The transmission coefficient to the transverse first excited state for the collision of two fermionic atoms in quasi-1D at $\epsilon = 1.5$ and $B = 199.793$ G (detuned by approximately -0.039 G from the free space $m = 0$ resonance at collision energy $2.5\hbar\omega$). All other parameters are the same as in Fig. 1. The coefficient reaches unity at $\alpha \approx 45.8°$ in this plot.

approach each other when ϵ approaches ϵ_c [22] from above, merge into one point at $\epsilon = \epsilon_c$, and disappear at $\epsilon < \epsilon_c$. When $\epsilon < \epsilon_c$ we always have $\mathcal{T} < 1$. When ϵ approaches 1 from above, $\mathcal{T} \propto (\epsilon - 1)^{1/2}$, consistent with the threshold law for inelastic collisions in 1D [23].

In conclusion, we have theoretically studied the low-dimensional scattering properties of a spin polarized Fermi gas near p-wave Feshbach resonances. Due to the spatial anisotropy of the p-wave interaction, the scattering properties are strongly dependent on the direction of the external magnetic field. We have also investigated the new inelastic scattering processes which are absent in previous theoretical models based on isotropic p-wave interactions. In particular, for the quasi-one-dimensional geometry, one can create an EPR entangled state of two identical fermions using the collision; this may be useful for quantum information with cold atoms.

S.-G. P. and K. J. are supported by NSFC (No. 11004224, No. 11204355, and No. 91336106), NBRP-China (No. 2011CB921601), CPSF (No. 2012M510187 and No. 2013T60762), and programs in Hubei province (No. 20130105010101024 and No. 2013CFA056). S. T. is supported by the NSF (Grant No. PHY-1068511) and by the Alfred P. Sloan Foundation. We thank Shangguo Zhu for discussions.

*shina.tan@physics.gatech.edu
†kjjiang@wipm.ac.cn

[1] C. Chin, R. Grimm, P. Julienne, and E. Tiesinga, Rev. Mod. Phys. **82**, 1225 (2010).
[2] C. Ticknor, C. A. Regal, D. S. Jin, and J. L. Bohn, Phys. Rev. A **69**, 042712 (2004).
[3] K. Günter, T. Stöferle, H. Moritz, M. Köhl, and T. Esslinger, Phys. Rev. Lett. **95**, 230401 (2005).
[4] B. E. Granger and D. Blume, Phys. Rev. Lett. **92**, 133202 (2004).
[5] L. Pricoupenko, Phys. Rev. Lett. **100**, 170404 (2008).
[6] J. Zhang, E. G. M. van Kempen, T. Bourdel, L. Khaykovich, J. Cubizolles, F. Chevy, M. Teichmann, L. Tarruell, S. J. J. M. F. Kokkelmans, and C. Salomon, Phys. Rev. A **70**, 030702 (2004).
[7] M. Olshanii, Phys. Rev. Lett. **81**, 938 (1998).
[8] E. Haller, M. Gustavsson, M. J. Mark, J. G. Danzl, R. Hart, G. Pupillo, and H. C. Nägerl, Science **325**, 1224 (2009); E. Haller, M. J. Mark, R. Hart, J. G. Danzl, L. Reichsöllner, V. Melezhik, P. Schmelcher, and H.-C. Nägerl, Phys. Rev. Lett. **104**, 153203 (2010).
[9] B. Fröhlich, M. Feld, E. Vogt, M. Koschorreck, W. Zwerger, and M. Köhl, Phys. Rev. Lett. **106**, 105301 (2011).
[10] S.-G. Peng, S. S. Bohloul, X.-J. Liu, H. Hu, and P. D. Drummond, Phys. Rev. A **82**, 063633 (2010); W. Zhang and P. Zhang, Phys. Rev. A **83**, 053615 (2011).
[11] S. G. Peng, H. Hu, X. J. Liu, and P. D. Drummond, Phys. Rev. A **84**, 043619 (2011); S. Sala, P.-I. Schneider, and A. Saenz, Phys. Rev. Lett. **109**, 073201 (2012); S. G. Peng, H. Hu, X. J. Liu, and K. J. Jiang, Phys. Rev. A **86**, 033601 (2012).
[12] Z. Idziaszek and T. Calarco, Phys. Rev. Lett. **96**, 013201 (2006).
[13] S. G. Peng, S. Q. Li, P. D. Drummond, and X. J. Liu, Phys. Rev. A **83**, 063618 (2011); S. G. Peng, X. J. Liu, H. Hu, and S. Q. Li, Phys. Lett. A **375**, 2979 (2011).
[14] L. B. Madsen, Am. J. Phys. **70**, 811 (2002).
[15] L. Pricoupenko, Phys. Rev. Lett. **96**, 050401 (2006).
[16] H. Bethe and R. Peierls, Proc. R. Soc. A **148**, 146 (1935).
[17] P. Massignan and Y. Castin, Phys. Rev. A **74**, 013616 (2006).
[18] One can evaluate $c_n(\epsilon)$ more easily by integrating the differential equations $c_x{}''(\epsilon) = \frac{3\Gamma(-(\epsilon/2))}{2\Gamma(1-(\epsilon/2))}$ and $c_z{}'(\epsilon) = -\frac{6\Gamma((1-\epsilon)/2)}{\Gamma(-(\epsilon/2))}$ with initial conditions $c_n(\epsilon) = (-2\epsilon - 1)^{3/2} + O(|\epsilon|^{-1/2})$ at $\epsilon \to -\infty$. We should analytically continue $c_n(\epsilon)$ toward greater values of ϵ by passing the isolated branch points from above. For instance, $\mathrm{Im} c_x(\epsilon) = 3\sqrt{\pi}\epsilon$ at $0 < \epsilon < 2$.
[19] W. C. Henneberger, Phys. Rev. A **22**, 1383 (1980); P. A. Maurone and T. K. Lim, Am. J. Phys. **51**, 856 (1983).
[20] $c_x^{1D}(\epsilon) = 3(\epsilon + 1)\zeta((1/2), (1 - \epsilon/2)) + 6\zeta(-(1/2), (1 - \epsilon/2))$ at $\epsilon < 1$, and $c_z^{1D}(\epsilon) = -12\zeta(-(1/2), -(\epsilon/2))$ at $\epsilon < 0$, where $\zeta(s, a)$ is the Hurwitz zeta function [21]. We should analytically continue $c_n^{1D}(\epsilon)$ toward greater values of ϵ by passing the isolated branch points from above.
[21] F. W. J. Olver, D. W. Lozier, R. F. Boisvert, C. W. Clark, *NIST Handbook of Mathematical Functions* (Cambridge University Press, Cambridge, England, 2010).
[22] For the trapping frequency in Ref. [3], using the values of v_{1m} and r_{1m} from Ref. [2], we find $\epsilon_c \approx 1.00224$.
[23] H. R. Sadeghpour, J. L. Bohn, M. J. Cavagnero, B. D. Esry, I. I. Fabrikant, J. H. Macek, and A. R. P. Rau, J. Phys. B **33**, R93 (2000).